Speckle phase averaging in high-resolution color holography

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Interest in wavelength multiplexing in holography derives naturally from the need for realistic color rendition as well as from resolution requirements. Imaging in coherent illumination is compromised by speckle. Speckle is prejudicial to quality, sharpness, contrast—in a word, to the fidelity of reproduction or holographic reconstruction. In incoherent light these patterns are canceled by spatial phase averaging. Incoherent light can indeed be regarded as a superposition of a very large number of coherent components whose phase factors are distributed at random. It is demonstrated that the averaging effect, ultimately caused by the law of large numbers, is achieved by the superposition of only three components, thus allowing simultaneously a true color rendition and an improvement in spatial resolution. The spatial statistical behavior of the amplitude of the sum of three intrinsically coherent waves, when they are incoherently superposed in an imaging system, is investigated. A random variable representing the amplitude of this sum is introduced. Then the cumulative probability function and the probability density function of the resulting amplitude are calculated. The white-light (infinite-wave illumination) case and the purely coherent (one-wave) case are analyzed. The results are interpreted with a heuristic vector model. [S0740-3232(97)00502-4]

1. INTRODUCTION

The information-carrying capacity of an optical system is subject to a diffraction limit of (Etendu)/λ^2. The Etendu takes into account not only the size of the source but also the angular aperture of the observing system. The amount of information that can usefully be fed into the source, therefore, depends on the mode of observation.[1,2]

Nevertheless, it is highly desirable not to feed into the input anything approaching the theoretical maximum of the information capacity. This means that the system has redundancy; that is, it is carrying the input information several or many times over in parallel channels. This in turn makes it resistant to noise, i.e., increases the fidelity of transmission. It is well known that coherent systems, which are frequently nonredundant, are easily upset by the presence of dust or small disturbing particles. There is only one way to ameliorate the image-forming fidelity: to increase the number of parallel channels (number of degrees of freedom), i.e., to increase the sampling capacity of the image-forming system.

Paraphrasing the famous parable of Eddington,[3,4] An ichthyologist should not be surprised, if exploring the ocean by a fishing net with a mesh size of 2 inches, to find that in the ocean there are no creatures smaller than 2 inches, nor should he be surprised, if using several nets simultaneously, to find things smaller than the smallest openings of all the nets used.

Any calculation concerning the resolution limit of an image is at best a guide to expected performance. Rather than attempting the task of defining resolution[5] in any image-forming system, we shall adopt the principle that the reciprocal of the image spot size resulting from a point source is the resolution limit in lines per millimeter.

In holographic precision imaging, i.e., microscopy[6–9] and endoscopy,[10] the spatial resolution is determined by speckle. Speckle is the elementary information cell of the image-forming system[11–13]; therefore it determines the system's information handling (sampling) capacity: It plays the role of the mesh sizes in Eddington's fishing nets. Finally, the information-handling capacity of an optical system is determined by the total number of the quanta of optical information (the total number of speckles). To increase the image fidelity, one should not try to eliminate the speckles. It is an impossible task, in any case, as the speckle-free beam is also information free, i.e., image free, and therefore useless. To increase the fidelity, image resolution, and contrast, one should increase the number of parallel channels and thus increase the density of speckle beyond the resolution of the detection system (the human eye, perhaps) to achieve speckle averaging.

For us to be able to investigate the nuclei of individual human tissue cells, details as small as 1 μm in size have to be resolved. The best resolution achieved until now in holographic endoscopy was of the order of 2.5 μm.[14]

Spatial averaging of speckle patterns in imaging with coherent light[15] can improve resolution as well as white-light reconstruction of image-plane microholograms.[14,16] One possibility for reducing the speckle noise in an image-plane hologram, regarded simply as a spatial carrier image, is to extend the recording source size,[17] i.e., to reduce the spatial coherence. Another possibility for increasing the resolution by speckle averaging is to record holograms with several wavelengths (thus in effect increasing the number of Eddington's fishing nets). The resolution of microscopic observation of a coherently illuminated object is indeed ameliorated by wavelength multiplexing.[18–21] The mere addition of the speckle patterns caused by different laser wavelengths[22,23] makes their averaging possible. This means that it would be possible to obtain both a higher resolution and a natural color reproduction[24] if the hologram were recorded with, for example, three laser lines of adequate wavelength (color recording).

Interest in wavelength multiplexing comes naturally from the need for realistic color rendition and directly from resolution requirements, as well. Wavelength multiplexing in color holography has already been studied from colorimetric, intensity, and diffraction-efficiency viewpoints. Colorimetric analysis has shown that a spectral sampling effect resulting from the use of a discrete number of laser lines that are spectrally narrow owing to coherence requirements has to be taken into account for matching true colors of an object. Peercy and Hesselink[25,26] discussed wavelength selection by investigating the sampling nature of the holographic process. During the recording of a color hologram, the chosen wavelengths point sample the surface-reflectance functions of the object. This sampling of color perception can be investigated by the tristimulus values of points in the reconstructed hologram, which is mathematically equivalent to integral approximations for the tristimulus integrals. According to Peercy and Hesselink, the sampling approach indicates that three monochromatic sources are almost always insufficient to preserve all of the object's spectral information accurately. Four, five, or more laser wavelengths may be required. Only further experiments for each specific case can determine the necessary number and the optimal combination of wavelengths. The other decisive factor that influences the choice of the recording wavelengths is their availability.

Intensity studies confirm that speckle averaging arises from wavelength variety, thus enhancing resolution. Diffraction-efficiency analysis[27–29] confirm that the problems of reciprocity law failures are managed best by simultaneous rather than sequential recording of all wavelengths. In any case, sequential exposure is impossible in pulsed recordings.

Here we shall show that the behavior of luminous radiance composed of three light waves of different wavelength is statistically closer to that of an incoherent than to that of a coherent light beam. We shall focus our attention on three geometrically identical waves, existing simultaneously but with mutually random phases. We shall study the behavior of the sum of these three components to see whether this sum takes certain particular values more frequently.

2. MODEL

The fact of interest is that the averaging of speckle noise is achieved by the superposition of only three waves. To explain it, a simplified one-dimensional analysis of the incoherent superposition of a finite number of coherent waves is presented. If the number of waves is very large (infinite), their superposition is tending toward its noncoherent limit.

Intrinsically (in themselves) coherent waves are designated here by index (j), that is, (ϕ_j), for example. The wave $\phi(x)$ can be written in complex notation as

$$\phi(x) = a(x)\exp[i\theta(x)], \tag{1}$$

or in a compact form,

$$\phi \equiv a \exp(i\theta), \tag{2}$$

$$\phi_j \equiv a_j \exp(i\theta_j). \tag{3}$$

Mutual noncoherence between intrinsically coherent waves $\phi_1, \phi_2, \phi_3, \ldots \phi_j$, etc., is treated by assuming that the corresponding $\theta_1, \theta_2, \theta_3, \ldots \theta_j$ are stochastically independent.

If certain illumination is composed of N mutually noncoherent waves $\phi_j(j = 1, 2, 3, \ldots N)$, then the image wave $\phi(x)$ of a single object point $\mathbf{x}$ is given as a superposition:

$$\phi(x) = \sum_{j=1}^{N} \phi_j(x) = \sum_{j=1}^{N} a_j \exp(i\theta_j). \tag{4}$$

The illumination intensity of the image point is then

$$I(x) = |\phi(x)|^2 = \left| \sum_{j=1}^{N} \phi_j(x) \right|^2, \tag{5}$$

and, according to Eq. (3),

$$I(x) = \left| \sum_{j=1}^{N} a_j \exp(i\theta_j) \right|^2. \tag{6}$$

A mere expansion of this gives

$$I(x) = \sum_{j=1}^{N} |a_j|^2 + 2\mathcal{R}\left\{ \sum_{j=1}^{N} \sum_{\substack{k=1 \\ j<k}}^{N} a_j a_k{}^* \exp[i(\theta_j - \theta_k)] \right\}, \tag{7}$$

where a_j and a_k are complex and $a_k{}^*$ stands for the complex conjugate of a_k.

In a shorter form,

$$I(x) = \sum_{j=1}^{N} |a_j|^2 + R, \tag{8}$$

R being

$$R = 2\mathcal{R}\left\{ \sum_{j=1}^{N} \sum_{\substack{k=1 \\ j<k}}^{N} a_j a_k{}^* \exp[i(\theta_j - \theta_k)] \right\}, \tag{9}$$

where $\mathcal{R}$ stands for the real part of the complex expression within the braces.

In noncoherent illumination imaging, R is neglected, as the summation is taken over a very large number $(N \to \infty)$ of incoherent waves. In the particular case of interest, N is always small ($N = 3$, 4, or 5, for example), and consequently this term also has to be taken into account.

Instantaneous phases $\theta_j, \theta_{j'}$ $(j, j' = 1, 2, 3, \ldots N)$ of the coherent light waves are characterized by a strong mutual correlation approaching 1 in the ideal coherent case. In that case they are simply proportional ($\theta_j = \mathbf{K}\theta_{j'}$) to each other.

In contrast, in the noncoherent case, the instantaneous phases $\theta_j, \theta_{j'}(j \neq j')$ would have uncorrelated random values.

3. THREE-WAVE MULTIPLEXING

In color holography, wavelength multiplexing takes place at the reconstruction. Usually three spectrally narrow (although broadened with respect to the laser lines used for recordings) light waves are emerging from each reconstructed image point.

Therefore $N = 3$, and the total wave amplitude $\phi(x)$ reads as

$$\phi(x) = \sum_{j=1}^{3} \phi_j = a_1 \exp(i\theta_1) + a_2 \exp(i\theta_2) + a_3 \exp(i\theta_3). \tag{10}$$

Of course, only the real part of this complex amplitude has a physical meaning and, therefore it has to be analyzed further. If it is denoted by S,

$$S = \mathcal{R}\{\phi(y)\} = a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3. \tag{11}$$

As θ_1, θ_2, and θ_3 are random variables, so is the sum S.

For a noncoherent superposition the phases θ_j are statistically independent, and they can embrace any value $(0 \leqslant \theta_j \leqslant 2\pi)$ at random and with no preferences. In other words, it is very likely that the probability $P(a \leqslant \theta_j \leqslant b)$ of encountering (finding) certain θ_j depends only on $(b - a)$. Owing to the normalization requirement,

$$P(a \leqslant \theta < b) = \frac{b - a}{2\pi}, \qquad 0 \leqslant a < b < 2\pi. \tag{12}$$

Each of amplitudes a_1, a_2, and a_3 is determined as a product of three factors: (1) the spectral reflectance function of the object that is holographed, (2) the relative intensities of three chromatic laser light components used for recording the hologram, and (3) the relative intensities of the same spectral components of the white-light reconstructing beam.

4. CUMULATIVE PROBABILITY FUNCTION OF THE SUM OF THREE WAVES

Since the random variables θ_1, θ_2, and θ_3 are stochastically independent, the cumulative probability function $\mathbf{F}_s(\mathbf{x})$ of the random variable S,

$$F_S(x) = P(S < x) = P(a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 < x), \tag{13}$$

can be expressed in the form of the integral

$$P(a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 < x) = \iiint_{a_1\cos\theta_1 + a_2\cos\theta_2 + a_3\cos\theta_3 < x} \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}\theta_3. \tag{14}$$

The integration domain of this integral, Fig. 1, is a volume in three-dimensional space with orthogonal reference frame $(\theta_1, \theta_2, \theta_3)$.

The volume of integration is bounded by a surface expressed by the equation

$$a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 = x. \tag{15}$$

If we change variables adequately,

$$u_j = \cos\theta_j, \qquad \text{and hence } -1 \leqslant u_j \leqslant 1, \tag{16}$$

the integration domain is then transformed into the simple form of a truncated cube (Fig. 2) centered at the origin and with one corner cut off by the plane:

$$a_1 u_1 + a_2 u_2 + a_3 u_3 = x. \tag{17}$$

Rewritten, the cumulative probability function is now

$$F(x) = P(a_1 u_1 + a_2 u_2 + a_3 u_3 < x); \tag{18}$$

that is,

$$F(x) = \iiint_{a_1u_1 + a_2u_2 + a_3u_3 < x} \frac{\mathrm{d}u_1}{\sqrt{1 - u_1^2}} \frac{\mathrm{d}u_2}{\sqrt{1 - u_2^2}} \frac{\mathrm{d}u_3}{\sqrt{1 - u_3^2}}$$

$$\text{since } \mathrm{d}\theta_j = \frac{\mathrm{d}u_j}{\sqrt{1 - u_j^2}}. \tag{19}$$

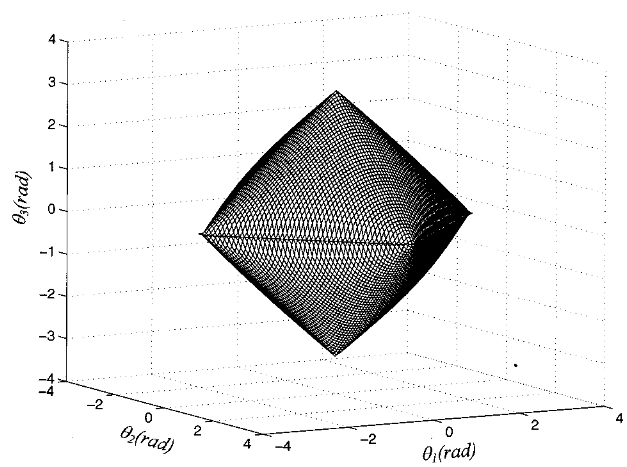

Fig. 1. Three-dimensional integration domain.

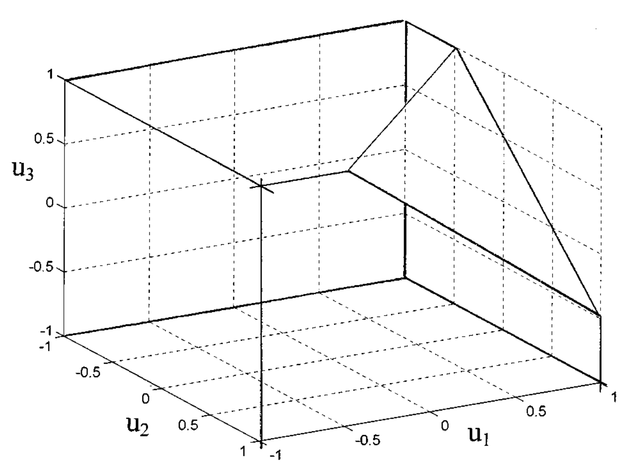

Fig. 2. Integration volume after a change of coordinates.

5. CALCULATIONS AND RESULTS

Integral (19) can be calculated by three successive one-dimensional integrations, but only the first one can be performed analytically by its primitive arcsine function. After this first integration is done, the cumulative probability function comes out in the form of an elliptical surface integral:

$$F(x) = \int\int \frac{\arcsin(h) + \pi/2}{\sqrt{1 - u_1^2}\sqrt{1 - u_2^2}} \mathrm{d}u_1 \mathrm{d}u_2, \qquad (20)$$

where h is the distance of the cutting plane P_y (varying with the sum $\mathbf{x}$) from the plane $u_3 = -1$.

$$h = \frac{x - a_1 u_1 - a_2 u_2}{a_3}. \qquad (21)$$

The remaining surface integral (20) can be calculated only by numerical procedures. If we apply the Gauss quadrature method,[30] for example, we obtain the cumulative probability density function $F(x)$ for each value (x), ranging from $x_{\min} = (-a_1 - a_2 - a_3)$ to $x_{\max} = (a_1 + a_2 + a_3)$. During this integration the cutting plane is successively truncating the cube of integration starting at its lower corner $(-1, -1, -1)$ and ending up at its upper corner located at (1, 1, 1). This can be imagined as a plane moving parallel to itself through the cube from the starting point $x_{\min}$ to the final position $x_{\max}$.

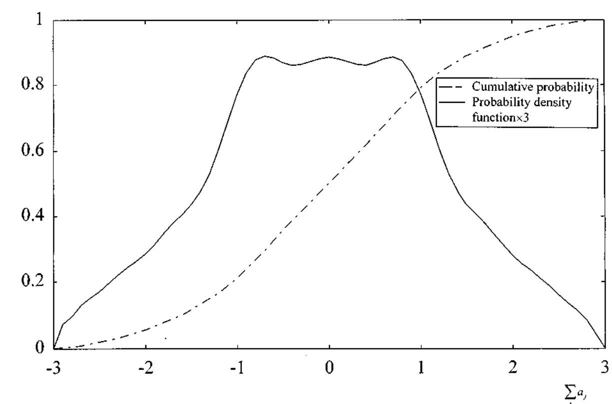

Fig. 3. Probability functions for $\{a_1 = a_2 = a_3 = 1.0\}$.

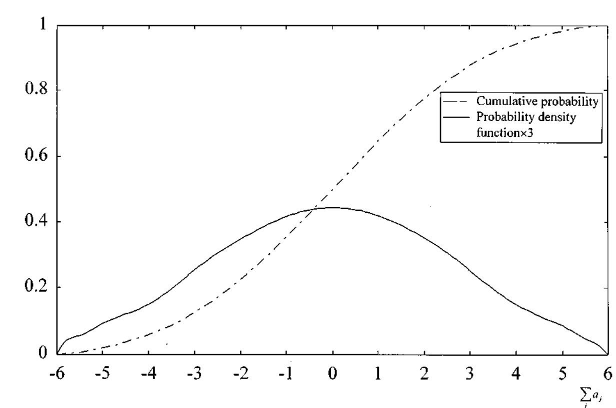

Fig. 4. Probability functions for $\{a_1 = 1.5, a_2 = 2.0, a_3 = 2.5\}$.

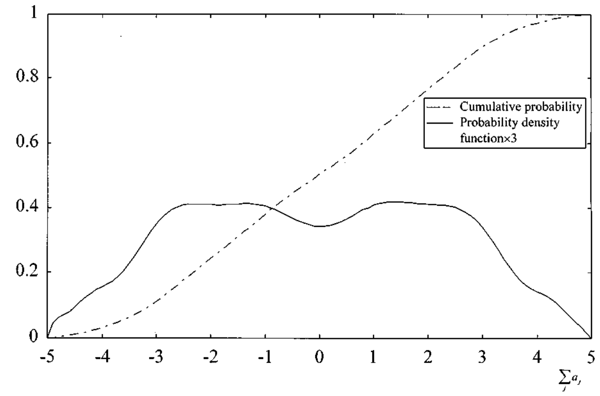

Fig. 5. Probability functions for $\{a_1 = 3.0, a_2 = 1.0, a_3 = 1.0\}$.

After the cumulative probability function is obtained, the corresponding probability density function is calculated by deriving the cumulative probability function. In practice this derivation is done by a stepwise finite-difference approximation. Obviously, probability functions are strongly dependent on the choice and combination of the initial parameters (a_1, a_2, a_3). Three plots of the probability functions for three different sets of amplitudes (a_1, a_2, a_3) are presented in Figs. 3, 4, and 5.

Finally, the range of the most probable values of the wave amplitude (11) of the non-coherent superposition of three waves emanating from each holographically reconstructed object point, could be estimated.

6. ANALYSIS AND INTERPRETATIONS

1. Let us assume that at first the three amplitudes (a_1, a_2, a_3) are mutually not very different from each other, for example, as in a scene with a large-band spectral reflectance function. In that case the most probable value for the resulting amplitude $(x = a_1 \cos \theta_1 + a_2 \cos \theta_2 + a_3 \cos \theta_3)$ [Eq. (15)] of the noncoherent superposition of three (chromatically) different waves is zero $(x = 0)$ (see Figs. 3 and 4). This represents the best possible three-wave averaging of the spatial phase variations (speckle). To reduce the speckle variations even more, (N) should be chosen larger than three: $(N > 3)$. The interesting question is, What is the minimum number of statistically independent (noncoherent, different color) waves that should be used for obtaining a given level of speckle noise reduction and consequently the spatial resolution improvement that approaches the desirable white-light limit?

2. In the case in which the condition $(a_1 \approx a_2 \approx a_3)$ of equal amplitudes is not satisfied, for example, $(a_1 > a_2 + a_3)$ (Fig. 5) the zero value is no longer the most probable one. In other words, the averaging over the random spatial phases is no longer so good.

3. To elucidate the two previous points, we can imagine the complex amplitude (a_j) of each wave as a vector in a complex space. Different vectors corresponding to different noncoherent waves are pointing toward random directions (to account for random phases), each having different (but comparable) constant lengths (correspond-

ing to wave amplitudes a_j). Obviously, the sum of several (three) vectors with random phases [each turning with constant angular speed ($2\pi\nu_j$) in a mutually independent, noncorrelated way] will become closer to zero as the vectors' lengths converge. The probability of a maximum constructive interference is not excluded completely but is very small.

7. CONCLUSIONS

Imaging in coherent illumination is compromised by the formation of diffraction patterns in a variety of forms and intensity distributions from Airy disks to general speckles.[31] Such patterns are prejudicial to the quality, sharpness, and contrast, that is, to the fidelity of the reproduction or holographic reconstruction. In incoherent light these patterns are canceled by spatial phase averaging. Incoherent light can indeed be regarded as a superposition of a very large number of coherent components whose phase factors are distributed at random. This averaging effect, ultimately caused by the law of large numbers, can be represented mathematically by the mean value of the complex sum

$$S = \frac{1}{N}\sum_{j=1}^{N} \exp(i\alpha_j), \tag{22}$$

where the α_j's are independent and uniformly distributed random variables. In this case the probability density of $|S|$ is approximately Gaussian[32] and zero centered, with a standard deviation of order $1/\sqrt{N}$. In other words, S will seldom deviate from zero for more than a few $1/\sqrt{N}$.

Consequently, when N is very large ($N \to \infty$), the probability of $|R|$ being greater than a given $\beta/\sqrt{N}$ is very low:

$$\mathrm{prob}\left(|R| > \frac{\beta}{\sqrt{N}}\right) \propto 10^{-f(\beta)}, \tag{23}$$

where $f(\beta)$ is an integer-valued function depending on the β chosen. In this case Eq. (8) becomes

$$\left|\sum_{j=1}^{N} \phi_j\right|^2 \cong \sum_{j=1}^{N} |a_j|^2. \tag{24}$$

A noncoherent superposition of a large number of intrinsically coherent waves is intensity based, behaving like a thermal source having a Gaussian intensity distribution.

Now the question is, as has already been pointed out, whether such an averaging effect occurs if a rather small number of coherent light components are superposed. In color holography seldom more than three different coherent components will be applied, and in the reconstruction they will overlap incoherently. In Fig. 6 a one-dimensional simulation of the spatial phase averaging effect is presented.

The first line represents an irregularly oscillating function that can be regarded as the intensity modulation in a typical speckle pattern. This speckle function is first arbitrarily shifted and then rescaled by a factor proportional to the wavelength ratios of the three-chromatic components. The mean value of the superpositions of the original speckle function with the first shifted and rescaled speckle function is shown as the second line of the Fig. 6. The third line represents the mean value of the original signal superposed on its first two shifted and rescaled versions, and so on. The law of large numbers ensures that the limit (far beyond the eighth line) will be flat: This would be the perfect state of averaging. Nevertheless, here the only aim is to demonstrate that the third line can be considered to have significantly less speckle than the first one.

The amplitudes of the original speckle function at given abscissas can be considered to be distributed at random if their mutual distances are larger than a few periods (wavelengths). This holds for any particular speckle distribution in one, two, or in three dimensions. If the three of these structures are superposed, the statistical frequencies of the deviations will be described by the probability density function [Eq. (11)] of the random variable S,

$$S = \frac{1}{3}(\cos\,\theta_1 + \cos\,\theta_2 + \cos\,\theta_3), \tag{25}$$

where θ_1, θ_2, θ_3 are uniformly distributed in the interval $[0, 2\pi]$.

The resulting probability density functions were presented in Figs. 3–5. One should observe that most of the deviations are confined within one half of the maximum, and this complies with the simulation curves (see Fig. 6). Fluctuations in the third line are half as large as the fluctuations of the original signal. But one should not forget that both the statistical model and the simulations are only heuristic, and they do not represent the real diffraction patterns in any sense. Moreover, even if they did, the mean values represented by the lower lines of Fig. 6 could eventually present only the structures (intensity patterns) recorded within the hologram (refractive-index modulation). But there is only an indirect and not at all simple relationship between the fidelity of the interference intensity pattern recordings and the fidelity of the holographic reconstruction. To find this relationship, one should perform a complete rigorous analysis of the reconstruction-beam diffraction at the complex three-dimensional modulation, and this is not an easy task.

The case of three multiplexed waves treated above consequently appears to lie between the following two extreme cases:

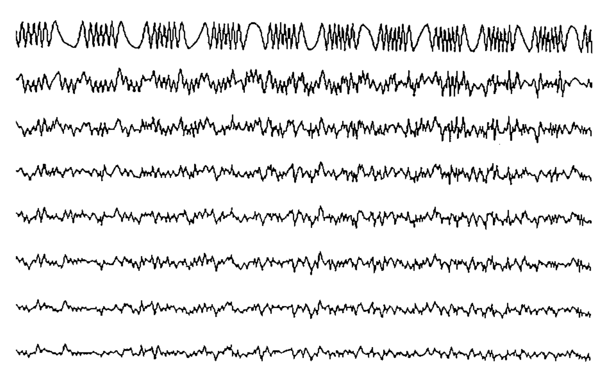

Fig. 6. Superposition of N random phase waves.

$-N = 1$, illumination by a single, coherent beam (e.g., a laser),

$-N \to \infty$, illumination by a noncoherent source (e.g., a white light).

In conclusion, the averaging of the diffraction structures achieved by the incoherent superposition of only three chromatic components of the color hologram reconstruction, is explained through an analysis that was based on a very simple model. In reality, the phases (θ_1, θ_2, θ_3, ... θ_j) are not random variables but random functions of coordinates. However, this does not affect the qualitative results.

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